



Measuring Risk Dependencies in the Solvency II-Framework

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Overview

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2. Dependency ratios
3. Copulas
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7. Conclusions

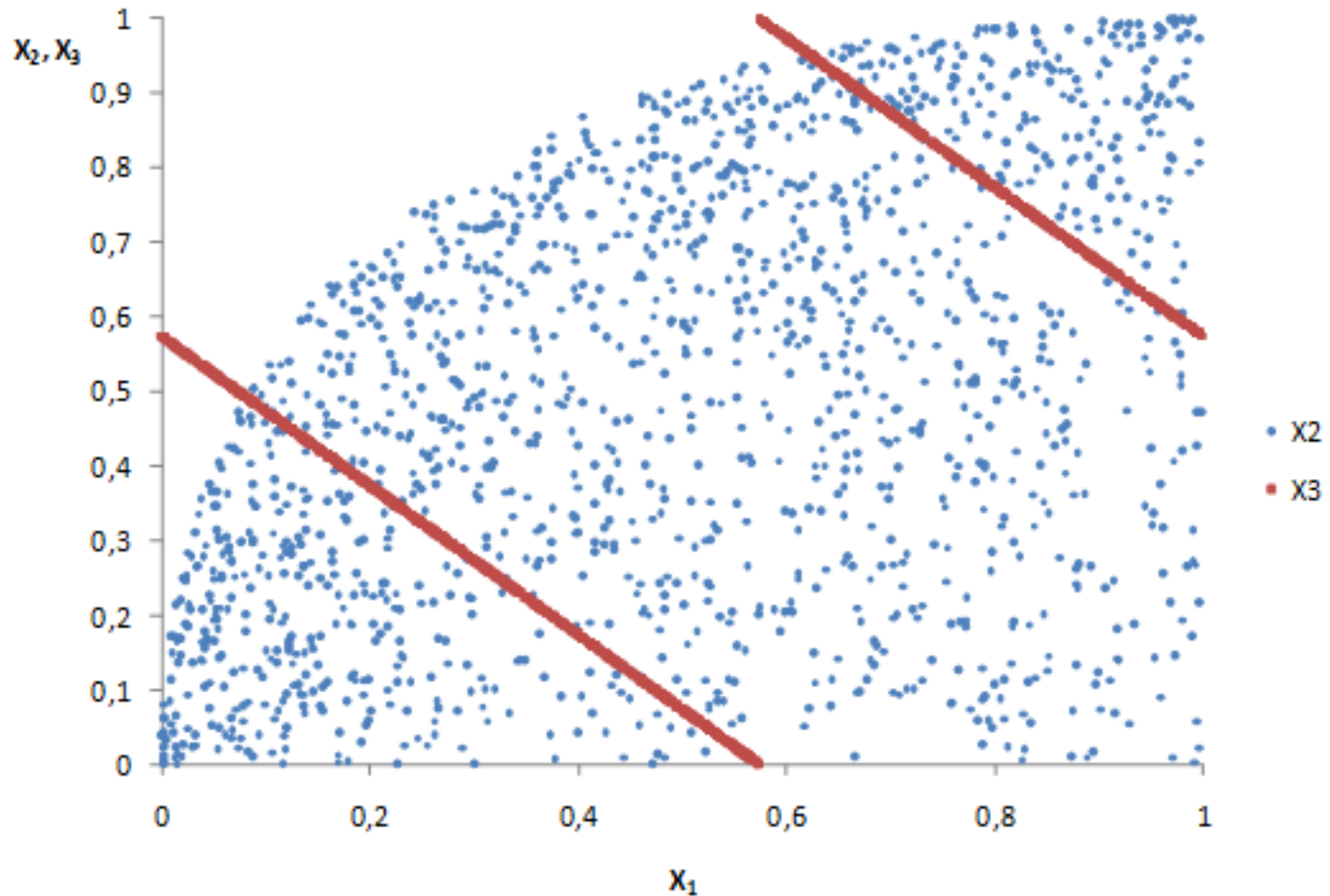
Introduction

- Solvency II obliges insurance undertakings to determine their overall loss distribution function
 - Necessity of considering dependencies between risks
 - Standard model Solvency II: linear correlations
 - Use of an internal model would allow the application of copulas
- Aim
 - Overview over the concept of copulas
 - Analyze and discuss their possible application in the Solvency II context
 - Assessment of the Solvency II directive with regard to the consideration of dependencies

Dependency ratios (1)

- Linear correlation coefficient
 - Formula: $\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X) * Var(Y)}}$
 - Advantages
 - Rudimentary, but simple
 - Describes dependencies in a single number
 - Shortcomings
 - Full correlation does not necessarily lead to $|\rho|=1$
 - No information on differences regarding the strength of dependencies across the range of values
 - High dependence though small amount of linear correlation coefficient is possible

Dependency ratios (2)



Dependency ratios (3)



- Spearman's rank correlation
 - Formula: $\rho_S(X, Y) = \rho(F_X(X), F_Y(Y))$
- Kendall's τ
 - Formula: $\tau(X, Y) = P((X_1 - X_2)(Y_1 - Y_2) > 0) - P((X_1 - X_2)(Y_1 - Y_2) < 0)$

Copulas

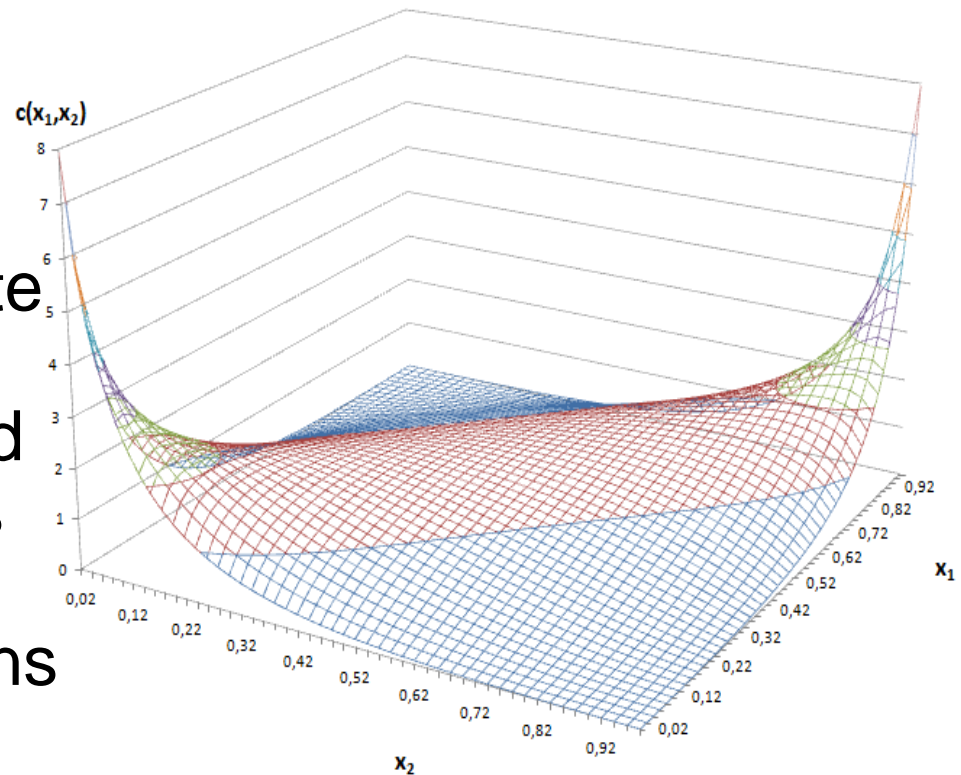


- Formula: $C(u_1, \dots, u_n) = P(U_1 \leq u_1, \dots, U_n \leq u_n)$
- Idea of copulas: separating the joint marginal distribution function into
 - one part that describes the dependence structure and
 - multiple parts that describe the marginal distribution functions
- Types of copulas
 - Elliptical copulas
 - Gaussian copulas
 - Student copulas
 - Archimedean copulas
 - Gumbel copulas
 - Cook-Johnson copulas
 - Frank copulas

Elliptical copulas – Gaussian copulas

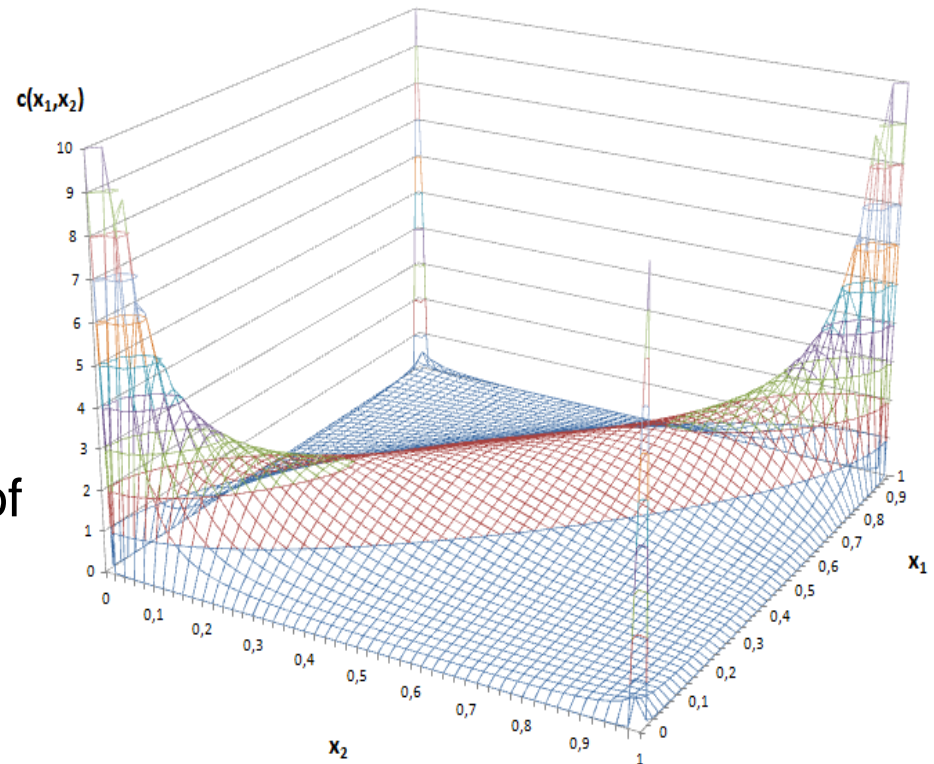


- Formula: $C_{\rho}^{Gau}(u_1, \dots, u_n) = \Phi_{\rho}^n(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_n))$
- Main Characteristics:
 - Dependency in the tails goes to zero
 - Leads to multivariate normal distribution functions if standard normal distributions are chosen as marginal distributions



Elliptical copulas – Student copulas

- Formula: $C_{v,\rho}^{Stu}(u_1, \dots, u_n) = t_{v,\rho}^n(t_v^{-1}(u_1), \dots, t_v^{-1}(u_n))$
- Main Characteristics:
 - No independency in the tails
 - Strength of dependence in the tail increases with
 - Decreasing degrees of freedom
 - Increasing correlation between the random variables

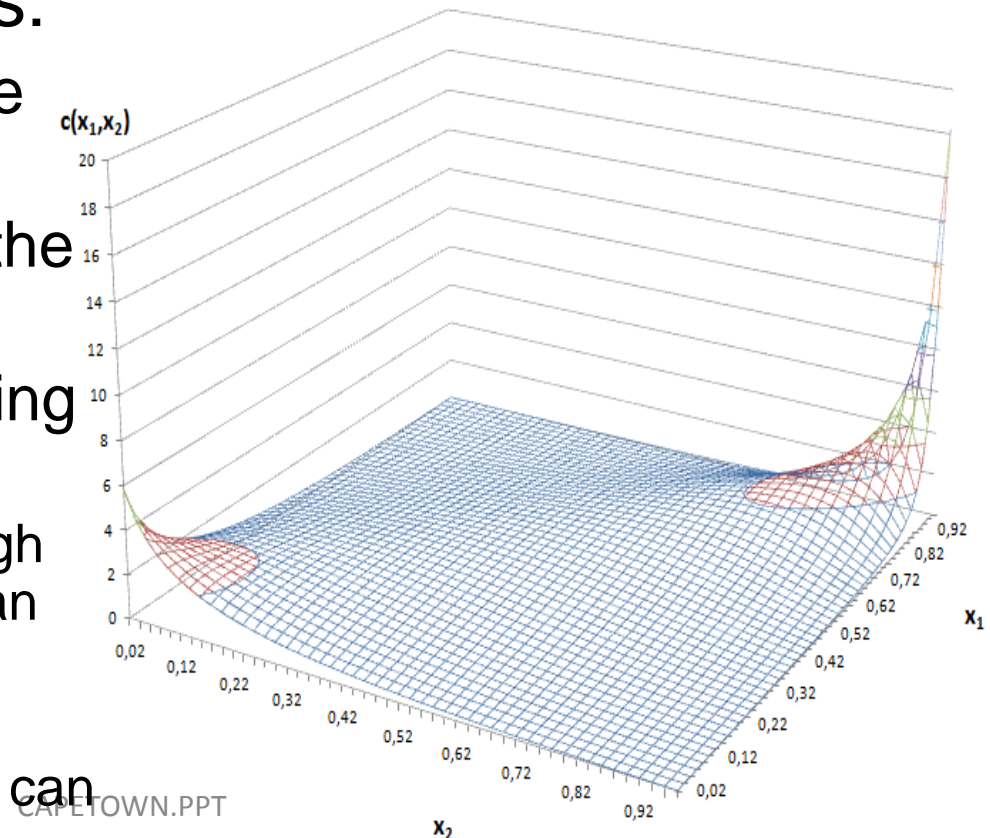


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Archimedean copulas – Gumbel copulas



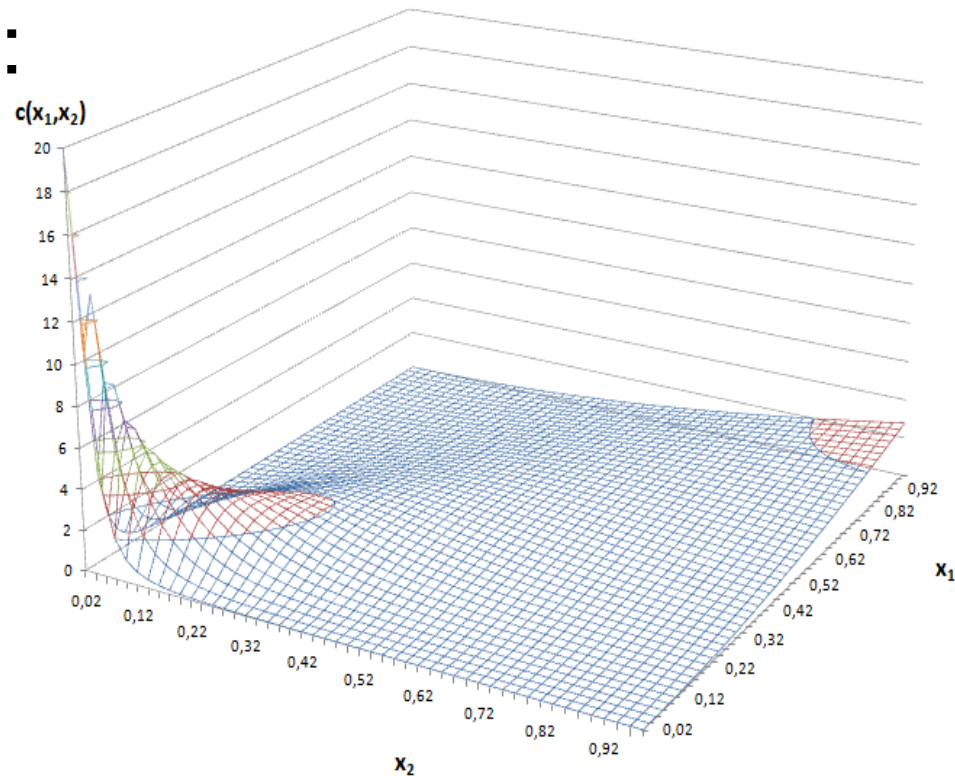
- Formula: $C_{\beta}^{Gum}(u_1, \dots, u_n) = e^{-\left(\sum_{i=1}^n (-\ln(u_i))^{\beta}\right)^{\frac{1}{\beta}}}$
- Main Characteristics:
 - Tail dependent in the upper tail
 - Tail independent in the lower tail
 - Adequate for modeling extreme events
 - Stress scenarios (high correlated losses) can be modeled
 - Common (lower independent) losses can be modeled



Archimedean copulas – Cook-Johnson copulas



- Formula: $C_{\beta}^{C-J}(u_1, \dots, u_n) = (u_1^{-\beta} + \dots + u_n^{-\beta} - n + 1)^{\frac{1}{\beta}}$
- Main Characteristics:
 - Tail dependent in the lower tail
 - Tail independent in the upper tail
 - Adequate for modeling yields on shares



Archimedean copulas – Frank copulas

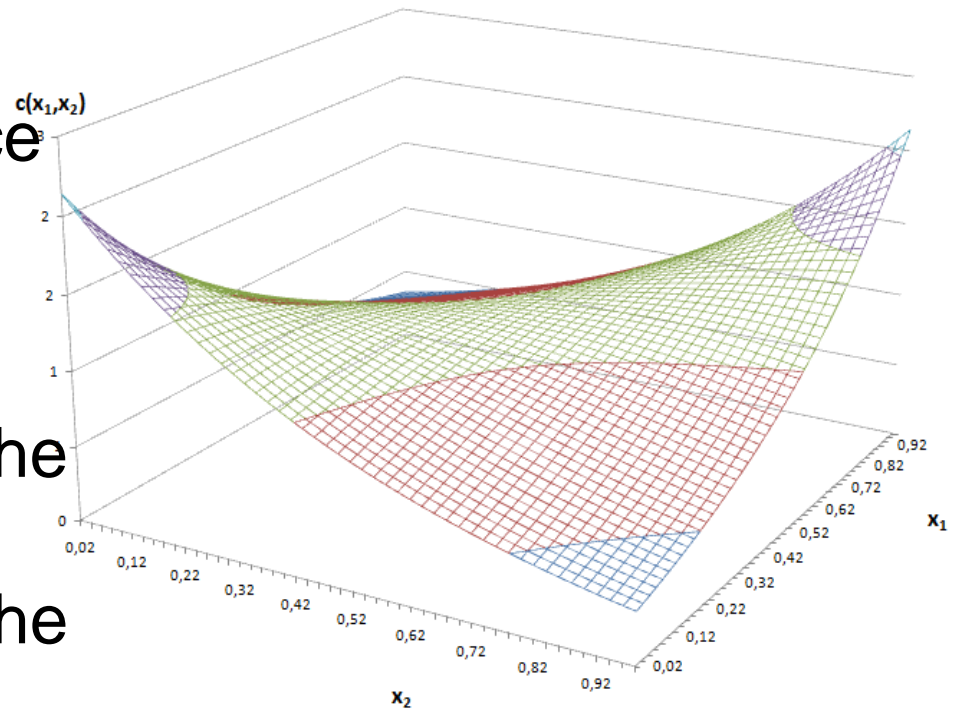


- Formula:
$$C_{\beta}^{Fra}(u_1, \dots, u_n) = -\frac{1}{\beta} \ln \left(1 + \frac{(e^{-\beta u_1} - 1) \dots (e^{-\beta u_n} - 1)}{(e^{-\beta} - 1)^{n-1}} \right)$$

- Main

Characteristics:

- Leads to dependence structures similar to those of Gaussian copulas
- Tail independent in the lower tail
- Tail independent in the upper tail



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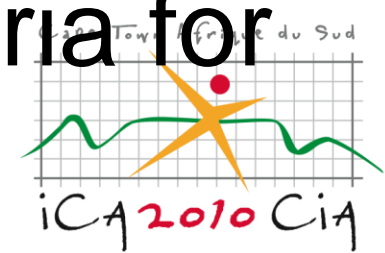
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Determination of copulas and multivariate distribution functions



- Two alternative approaches
 - Parametric approach
 - Choose a type of copula (use f. ex. procedure established by *Genest, C./Rivest, L.-P. (1993)*)
 - Determine parameters in such a way that the dependence structure given by the copula best fits to the observations (use f. ex. maximum likelihood method)
 - Non-parametric approach
 - Determine an empirical copula (not determining a specific type of copula in advance; empirical copula converges to the actual copula with increasing number of observations)
 - Empirical multivariate distribution function is given by this empirical copula and the empirical marginal distribution functions

Assessment – desirable criteria for dependency ratios



- $\delta(\cdot)$ is a dependency ratio
 1. symmetry: $\delta(X, Y) = \delta(Y, X)$
 2. standardization: $-1 \leq \delta(X, Y) \leq 1$
 3. conclusion based on and on co- and countermonotonicity
 - a. $\delta(X, Y) = 1 \Leftrightarrow X, Y$ are comonotone
 - b. $\delta(X, Y) = -1 \Leftrightarrow X, Y$ are countermonotone
 4. Invariance with regard to strictly monotone transformations: For a transformation $T: \mathbb{R} \rightarrow \mathbb{R}$ strictly monotone on the codomain of X the following holds:
 - a. $\delta(T(X), Y) = \delta(X, Y)$, if T is strictly monotonic increasing
 - b. $\delta(T(X), Y) = -\delta(X, Y)$, if T is strictly monotonic decreasing
 5. conclusion based on and on independence
$$\delta(X, Y) = 0 \Leftrightarrow X, Y \text{ are independent}$$

Assessment – introduced concepts (1)



- Dependency ratios
 - Pearson linear correlation coefficient
 - Fulfils criterion 1. and 2.
 - Only defined for finite variances of the random variables
 - Spearman's rank correlation
 - Fulfils criterion 1., 2., 3. and 4.
 - Does not only measure linear dependency, but also the monotone dependency in common
 - Kendall's τ
 - Fulfils criterion 1., 2., 3. and 4.
 - Does not only measure linear dependency, but also the monotone dependency in common

Assessment – introduced concepts (2)



- Copulas
 - Can be used to model multivariate distributions which fully describe the dependence structure
 - Particularly if dependencies are not linear, but are located in the tails, risk could be significantly underestimated if f. ex. the linear correlation coefficient is used
 - Copulas allow separate modeling of marginal distributions and dependence structure between them
 - Copulas allow multivariate distribution functions with marginals that are of different types
 - Copulas are invariant with regard to strictly monotone transformations

Assessment – shortcomings of copulas



- Dependency ratios have advantages in their practical usage
 - they are much easier to calculate
 - require less effort
 - much more experience exists with regard to their application in insurance companies
- Even though copulas lead to clearly superior results with regard to the quality of estimating dependencies between risks, copulas do not describe the direction in which one random variable affects another
- Copulas require high amount of data for modeling

Analysis of the consideration of risk dependencies in Solvency II



- Requirement: determination of SCR → determination of the *overall* loss distribution function
- Solvency II directive proposes aggregation of the so-called “risk modules” via linear correlation in the standard model
- Solvency II directive even states clear numbers to be used for aggregating the risk modules

Analysis of the consideration of risk dependencies in Solvency II (2)



- Basic Solvency Capital Requirement, the capital requirement for operational risk and the adjustment for the loss-absorbing capacity of technical provisions and deferred taxes are simply added (assumes full dependence)
- At least in the calculation of the BSCR dependencies are recognized, however:
 - Given correlations seem to be highhanded and do not reflect the specific situation of an individual company
 - Weaknesses of linear correlations have been shown
- Insurers have the possibility or can even be requested to recognize dependencies in a more sophisticated way, if they use an internal model:
 - Other dependency ratios might be applied
 - Copulas might be applied

Conclusions



- Insurers which measure the dependencies between their risks in a more sophisticated way should be rewarded by reducing the SCR or the other way around by imposing higher requirements on companies which use the rudimental standard approach
- It would also make sense and give additional incentives to explicitly mention the concept of copulas in the directive
- If correlations are used, at least no highhanded values should be used

Thank you!



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